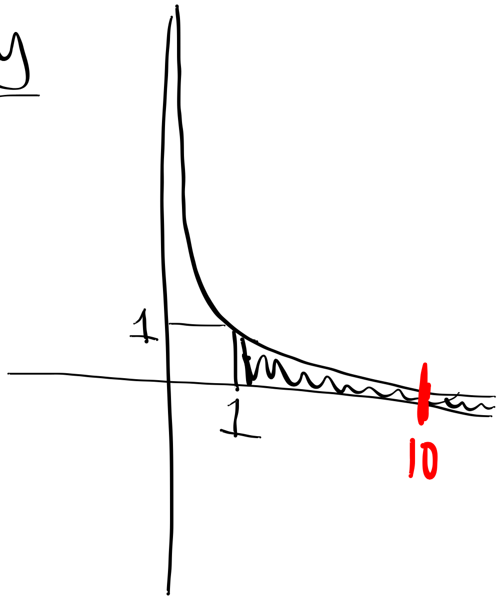
Integrals & Infinity

$$\int_1^{\infty} \frac{1}{x^2}\,dx$$



$$\int x^{-2}\,dx = \frac{x^{-1}}{-1}$$

$$\int_1^{10} \frac{1}{x^2}\,dx = -\frac{1}{10} - (-1) = \frac{9}{10}$$

$$\int_1^{100} \frac{1}{x^2}\,dx = \frac{99}{100}$$

Improper Integral

def

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^2}\,dx$$

$$= \lim_{b\to\infty}\left(-x^{-1}\right)\Big|_1^b = \lim_{b\to\infty} -b^{-1} - \left(-1^{-1}\right)$$

$$= \lim_{b\to\infty} -\frac{1}{b} + 1 = 1$$

Feb 13-7:39 AM

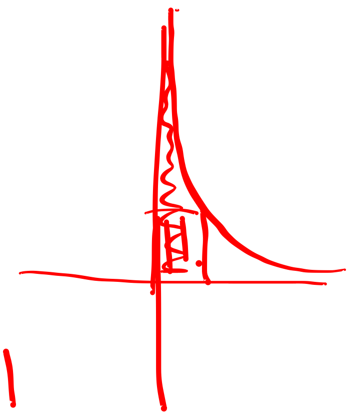
Now....
$$\int_1^\infty \frac{1}{x}\,dx =$$

$$\lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx = \lim_{b\to\infty} \ln(x)\Big|_1^b$$

$$= \lim_{b\to\infty} \ln(b) - \ln(1) = +\infty$$

(dne)

$$\int_0^1 \frac{1}{x^2}\,dx =$$



$$\lim_{a\to 0}\int_a^1 \frac{1}{x^2}\,dx = \lim_{a\to 0} -\frac{1}{x}\Big|_a^1$$

$$= \lim_{a\to 0} -\frac{1}{1} - \left(-\frac{1}{a}\right) = \lim_{a\to 0}\frac{1}{a} - 1$$

$$= +\infty$$

$$\int_1^\infty \frac{1}{(x-3)^2}\,dx =$$

$$\int_1^3 \frac{1}{(x-3)^2}\,dx + \int_3^{10} \frac{1}{(x-3)^2}\,dx + \int_{10}^\infty \frac{1}{(x-3)^2}\,dx$$

2 "problems"
or, rather,
2 improprieties

* V.A @ $x=3$
* ∞ in the upper limit of integration

$$\lim_{b\to 3} \int_1^b (x-3)^{-2}\,dx = \lim_{b\to 3} \left.\frac{-1}{x-3}\right|_1^b$$

$$= \lim_{b\to 3^-} \frac{-1}{b-3} - \left(\frac{-1}{1-3}\right) = \lim_{b\to 3^-} -\frac{1}{2} - \frac{1}{b-3} = +\infty$$

Feb 13-8:08 AM

8.2/9

$\int udv = uv - \int vdu$

$$\int \sqrt{x}\,\ln x\,dx$$

$u = \ln x \qquad dv = x^{1/2}\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{2x^{3/2}}{3}$

$$\frac{2x^{3/2}}{3}\ln x - \frac{2}{3}\int x^{3/2}\left(\frac{1}{x}\right)dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\left(\frac{2}{3}x^{3/2}\right) + C$$

Feb 13-8:21 AM

8.2/4)

$$\int x^2 e^{-2x}\,dx =$$

$u = x^2 \qquad dv = e^{-2x}\,dx$

$du = 2x\,dx \qquad v = -\frac{1}{2}e^{-2x}$

$$= -\frac{1}{2}x^2 e^{-2x} - \left(-\frac{1}{2}\int 2x e^{-2x}\,dx\right)$$

$$= -\frac{1}{2}x^2 e^{-2x} + \int x e^{-2x}\,dx$$

$u = x \qquad dv = e^{-2x}\,dx$

$du = dx \qquad v = -\frac{1}{2}e^{-2x}$

$$= -\frac{1}{2}x^2 e^{-2x} + \left[-\frac{1}{2}xe^{-2x} - \int \left(-\frac{1}{2}\right)e^{-2x}\,dx\right]$$

$$= -\frac{1}{2}x^2 e^{-2x} - \frac{1}{2}xe^{-2x} + \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\right) + C$$

$\int u\,dv = uv - \int v\,du$

$\int e^{-2x}\,dx$

$u = -2x$

$du = -2\,dx$

$-\frac{1}{2}du = dx$

$-\frac{1}{2}\int e^u\,du$

$= -\frac{1}{2}e^u + C$

$= -\frac{1}{2}e^{-2x}$

$$\int_0^1 xe^x\,dx =$$

$\int u\,dv = uv - \int v\,du$

$u = x \qquad dv = e^x\,dx$

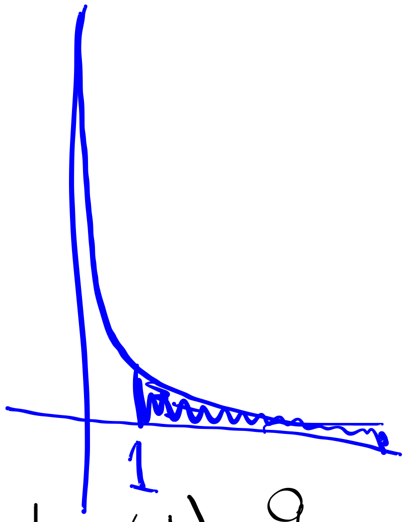
$du = dx \qquad v = e^x$

$$\left(xe^x\right)\Big|_0^1 - \int_0^1 e^x\,dx = xe^x\Big|_0^1 - e^x\Big|_0^1$$

$$= (e - 0) - (e^1 - e^0) = 1$$

Integrals and Infinity

$$\int_1^{\infty} \frac{1}{x^2}\,dx$$



$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1}+C = -\frac{1}{x}+C$$

$$\int_1^{10} \frac{1}{x^2}\,dx = \left(-\frac{1}{x}\right)\Big|_1^{10} = -\frac{1}{10} - \left(-\frac{1}{1}\right) = \frac{9}{10}$$

$$\int_1^{100} \frac{1}{x^2}\,dx = \left(-\frac{1}{x}\right)\Big|_1^{100} = -\frac{1}{100} - (-1) = \frac{99}{100}$$

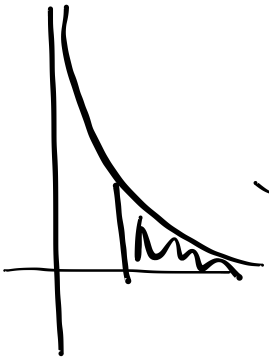
$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

[[improper integrals]]

$$\int_1^\infty \frac{1}{x^2}dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^2}dx$$

$$= \lim_{b\to\infty}\left(-\frac{1}{x}\right)\Big|_1^b = \lim_{b\to\infty} -\frac{1}{b} - (-1) = +\underline{1}$$

Feb 13-11:25 AM



$$\int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty} \ln(x)\Big|_1^b = \lim_{b\to\infty} \ln(b) - \ln(1)$$

$$= +\infty$$

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$$\int_0^1 \frac{1}{x}\,dx = \lim_{a \to 0} \int_a^1 \frac{1}{x}\,dx = \lim_{a \to 0} (\ln x)\Big|_a^1$$

$$= \lim_{a \to 0} \ln(1) - \ln(a)$$

$$= +\infty$$

$\therefore$ does not exist

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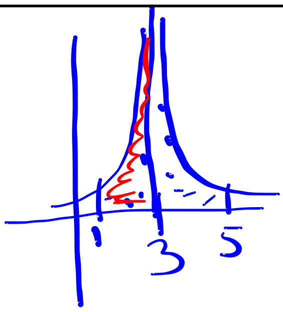

$$\int_1^5 \frac{1}{(x-3)^2}\,dx =$$

$$\int_1^3 \frac{1}{(x-3)^2}\,dx + \int_3^5 \frac{1}{(x-3)^2}\,dx$$

$$= \lim_{b\to 3^-} \int_1^b \frac{1}{(x-3)^2}\,dx + \lim_{a\to 3^+} \int_a^5 \frac{1}{(x-3)^2}\,dx$$

$$= \lim_{b\to 3^-} \frac{-1}{(x-3)}\Bigg|_1^b = \lim_{b\to 3^-} \frac{-1}{b-3} - \left(\frac{-1}{1-3}\right) = +\infty$$

$\therefore$ the entire Imp. Int. dne

$$\int_0^1 xe^x\,dx =$$

$$\int u\,dv = uv - \int v\,du$$

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$= xe^x\Big|_0^1 - \int_0^1 e^x\,dx$$

$$= (e^1 - 0) - (e^1 - e^0) = 1$$

Feb 13-12:03 PM